2.3

Polynomial Functions of Higher Degree with Modeling

Polynomial functions of higher degree include **cubic functions** (degree 3) and **quartic functions** (degree 4). Remember polynomials have the form: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x + a_0$.

Vocabulary of Polynomials

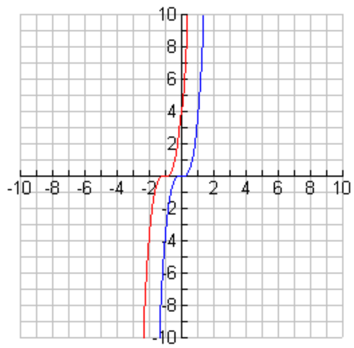
- Each monomial in this sum $a_n x^n$, $a_{n-1} x^{n-1}$, … $a_1 x$, …a_0 is a **term** of the polynomial.
- A polynomial function written in this way, with terms in descending degree, is written in **standard form**.
- The constants $a_n, a_{n-1}, \dots a_0$ are the **coefficients** of the polynomial.
- The term $a_n x^n$ is the **leading term**, and a_0 is the constant term.

Graphing transformations of Monomial Functions:

See example 1 on pg. 200.

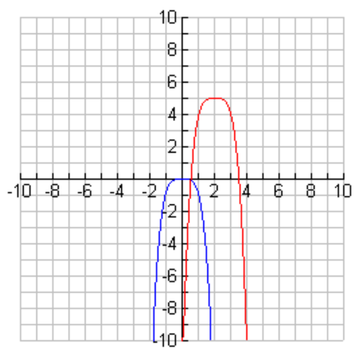
a) $g(x) = 4(x+1)^3$

Find y-int.



b) $h(x) = -(x-2)^4 + 5$

Find y-int.

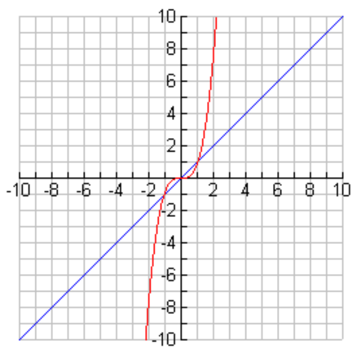


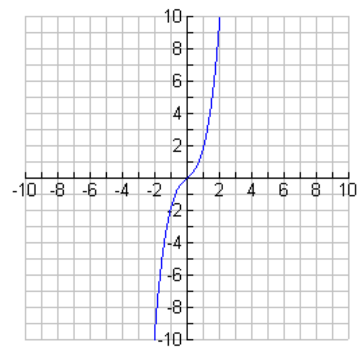
Graphing Combinations of Monomial Functions

Example 2 pg. 201

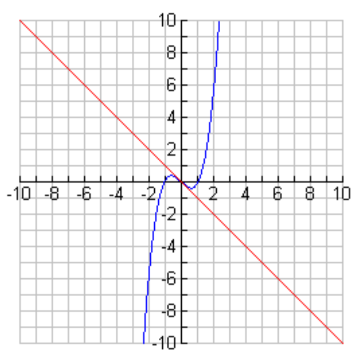
Graph the polynomial function, locate its extrema and zeros, and explain how it is related to the monomials from which it was built.

a) $f(x) = x^3 + x \quad \rightarrow$ factors to $x(x^2+1)$





b) $g(x) = x^3 - x$



Local Extrema and Zeros of Polynomial Functions

A polynomial function of degree *n* has at most ***n* - 1** local extrema and at most ***n*** zeros.

See Example 3 pg. 203.

Characteristic of Polynomial function is **End Behavior is closely** related to the end behavior of the leading term. (See pg. 204)

See example 4 and 5 on pgs. 204- 205.

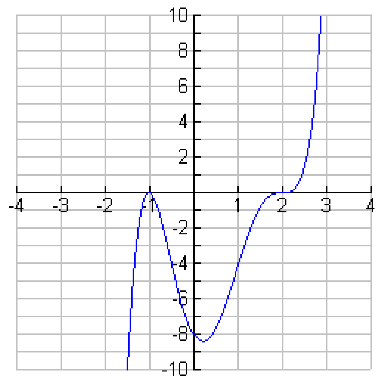
<u>Multiplicity of a zero of a polynomial function</u>:

If *f* is a polynomial function and $(x - c)^m$ is a factor of *f* , but $(x - c)^{m+1}$ is not, then *c* is a zero of **multiplicity *m*** of *f*.

A zero of multiplicity m ≥ 2 is a **repeated 0.**

$$f(x) = (x-2)^3(x+1)^2$$

Zeros at -1 and 2. Graph touches at -1 (mult. 2 even will just touch) graph crosses at 2 (mult. 3 odd will cross).



If a polynomial function *f* has a real zero *c* of odd multiplicity, then the graph of *f* crosses the *x*-axis at (*c*,0) and the value of *f* changes sign at *x* = *c*.

If the polynomial function *f* has a real zero *c* of even multiplicity, then the graph of *f* does not cross the *x*-axis at (*c*,0) and the value of *f* does not change sign at *x* = *c*.

If the polynomial function *f* has a real zero *c* of even multiplicity then the graph of *f* touches the *x*-axis at *x* = *c*.

Sketch the graph of $f(x) = (x-3)^3(x+2)^2$.

Using only algebra, find the cubic function with the given zeros.

a) 3, -2, 1

b) -2 (multiplicity 2) and -1

<u>Intermediate Value Theorem</u> – if there is a sign change from one value to another value between an interval then the sign change implies a real zero in that interval.

See example 7 on pg. 206